TKN/KS/16/5757

Bachelor of Science (B.Sc.) Semester-I (C.B.S.) Examination

MATHEMATICS (M_2-Calculus)

Compulsory Paper—II

Time—Three Hours] [Maximum Marks—60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Question No. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) By using (ϵ-δ) definition of limit of the function at a point, show that :

$$\lim_{x \to 1} (x^2 + 3) = 4.$$ 6

(B) Examine the continuity of the function f(x) at a point x = a, where :

$$f(x) = \begin{cases} \dfrac{x^2}{a} - a & \text{for } x \le a \\ a - \dfrac{x^2}{a} & \text{for } x > a. \end{cases}$$ 6

OR

(C) Discuss the derivability of the function :

$$f(x) = \begin{cases} x, & x < 1 \\ 2 - x, & 1 \le x \le 2 \\ -2 + 3x - x^2, & x > 2 \end{cases}$$

at x = 1, 2. 6

(D) If $y = a\cos(\log x) + b\sin(\log x)$, then prove that $x^2y_{n+2} + (2n+1)x\,y_{n+1} + (n^2+1)\,y_n = 0.$ 6

UNIT—II

2. (A) By applying Maclaurin's theorem, prove that :

$$e^x \cos x = 1 + x - \frac{2x^3}{3!} - \frac{2^2 x^4}{4!} - \frac{2^2 x^5}{5!} + \frac{2^3 x^7}{7!} + \ldots\ldots$$

6

(B) Prove that the radius of curvature at any point of the catenary $y = c\cosh(x/c)$ varies as the square of the ordinate. 6

OR

(C) Find all the asymptotes of the curve :

$$y^3 - 6xy^2 + 11x^2y - 6x^3 + x + y = 0.$$ 6

(D) Find :

$$\lim_{x \to 0} \left(\frac{\tan x}{x}\right)^{1/x}.$$ 6

UNIT—III

3. (A) If $z(x + y) = x^2 + y^2$, then show that :

$$\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right).$$ 6

(B) Prove that if $z = f(x, y)$ is a homogeneous function of degree n, then :

$$x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} = n(n-1)z.$$ 6

OR

(C) Verify Euler's theorem for :

$$Z = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right).$$ 6

(D) If $u_1 = \frac{x_2 x_3}{x_1}, u_2 = \frac{x_3 x_1}{x_2}$ and $u_3 = \frac{x_1 x_2}{x_3}$ then prove that :

$$J(u_1, u_2, u_3) = 4.$$ 6

UNIT—IV

4. (A) Evaluate :

$$\int \frac{x^3 + 3}{\sqrt{x^2 + 1}} dx.$$ 6

(B) Show that :

$$\int_1^2 \frac{dx}{(x+1)\sqrt{x^2 - 1}} = \frac{1}{\sqrt{3}}.$$ 6

OR

(C) Prove the reduction formula :

$$\int \cot^n x \, dx = -\frac{\cot^{n-1} x}{n-1} - \int \cot^{n-2} x \, dx$$

Hence evaluate $\int \cot^3 x \, dx$. 6

(D) Show that :

$$\int_0^\pi \frac{x \sin x}{1 + \cos^2 x} dx = \frac{\pi^2}{4}.$$ 6

UNIT—V

5. (A) Examine the continuity of the function defined by $f(x) = \frac{|x-a|}{(x-a)}, x \neq a$ and $f(x) = 1$, $x = a$ at the point $x = a$. 1½

(B) Show that $f(x) = x\,|x|$ is derivable at the origin. 1½

(C) Expand $\log(x + a)$ in powers of x by Taylor's theorem. 1½

(D) Determine $\lim\limits_{x \to 0} \dfrac{e^x - e^{-x} - 2\log(1+x)}{x \sin x}$. 1½

(E) If $u = \sin^{-1}\left(\dfrac{x^2 + y^2}{x + y}\right)$, show that :

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u.$$ 1½

(F) If $u = x^2 - y^2$, where $x = 2r - 3s + 4$, $y = -r + 8s - 5$, find $\dfrac{\partial u}{\partial r}$. 1½

(G) Show that :

$$\int_0^{\pi/2} \frac{\sqrt{\sin x}\,dx}{\sqrt{\sin x} + \sqrt{\cos x}} = \frac{\pi}{4}.$$ 1½

(H) Evaluate :

$$\int_0^{\pi/2} \sin^5 x \cos^8 x \, dx.$$ 1½